$\frac{dy}{dx} = e^{\frac{x-y}{2}}$ $\frac{dy}{dx} = -e^{\frac{y-x}{2}}$

$\left(e^{\frac{x-y}{2}}\right)\left(-e^{\frac{y-x}{2}}\right) = -e^{\frac{x-y}{2}+\frac{y-x}{2}} = -e^0 = -1$

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65 c) $\frac{dy}{dx} = x - \frac{1}{x^2}$

$f(x) = \begin{cases} \frac{1}{x} + \frac{x^2}{2} + c_1, & x<0 \quad (b) \\ \frac{1}{x} + \frac{x^2}{2} + c_2, & x>0 \quad (a) \end{cases}$

(a) $y(1)=2$ $x=1$ $y=2$ $c_2 = \frac{1}{2}$

(b) $y(-1)=1$ $x=-1$ $y=1$ $c_1 = \frac{3}{2}$

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6.2a Integration by Substitution

A change of variables can turn an unfamiliar integral into one that we can evaluate. (The differential matters.)

$\int f(x)\,dx = \int g(u)\,du$

$\int \sin(x) e^{\cos(x)} dx$ let $u = \cos x$ inside of composite

$\int \cancel{\sin x}\, e^u \frac{du}{\cancel{-\sin x}}$ $\frac{du}{dx} = -\sin x$

$du = -\sin x\, dx$

$-du = \sin x\, dx$

$\frac{du}{-\sin x} = dx$

$-\int e^u du$

$-e^u + c$

$\boxed{-e^{\cos x} + c}$

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$\int x^2 \sqrt{5+2x^3}\, dx$ let $u = 5+2x^3$

$\int \cancel{x^2} \sqrt{u} \frac{du}{6\cancel{x^2}}$ $\frac{du}{dx} = 6x^2$

$\frac{1}{6}\int \sqrt{u}\, du$ $du = 6x^2 dx$

$\frac{du}{6x^2} = dx$

$\frac{1}{6}\int u^{\frac{1}{2}} du$

$\frac{1}{6} \cdot \frac{u^{3/2}}{3/2} + C$ RTC Dudes

$\frac{1}{6} \cdot \frac{2}{3}\left(5+2x^3\right)^{3/2} + c$

$\frac{1}{9}\sqrt{(5+2x^3)^3} + c$

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$\int \cot(7x)\,dx$

$\int \frac{\cos(7x)}{\sin(7x)}\,dx$ $\quad u = \sin(7x)$

$\int \frac{\cancel{\cos(7x)}}{u} \frac{du}{7\cancel{\cos(7x)}}$ $\quad \frac{du}{dx} = 7\cos(7x)$ Find dx

$du = 7\cos(7x)\,dx$

$\frac{1}{7}\int \frac{1}{u}\,du$ $\quad \frac{du}{7\cos(7x)} = dx$

$\frac{1}{7}\ln|u| + C$

$\frac{1}{7}\ln|\sin(7x)| + C$

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$\int \frac{dx}{\cos^2 2x}$

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$\int \cot^2 3x\,dx$

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$\int \cos^3 x\,dx = \int (\cos x)^3\,dx$

$\sin^2 x + \cos^2 x = 1$

$\int \cos x \cos^2 x\,dx$

$\cos^2 x = 1 - \sin^2 x$

$\int \cos x\,(1 - \sin^2 x)\,dx$

$u = \sin x$

$\int (1 - u^2)\,du$

$\frac{du}{dx} = \cos x$

$du = \cos x\,dx$

$u - \frac{u^3}{3} + C$

$\sin x - \frac{\sin^3 x}{3} + C$

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Definite Integrals

$\int_0^{\frac{\pi}{3}} \tan x \sec^2(x)\,dx$

$x=\frac{\pi}{3}$ $u=\tan\frac{\pi}{3}=\sqrt{3}$

$x=0$ $u=\tan 0=0$

$u=\tan x$

$\frac{du}{dx}=\sec^2 x$

$du=\sec^2 x\,dx$

$\int_0^{\sqrt{3}} u\,du$

$\frac{u^2}{2}\Big|_0^{\sqrt{3}}$

$\frac{(\sqrt{3})^2}{2}-\frac{0^2}{2}=\frac{3}{2}$

$\int u\,du$

$\frac{u^2}{2}$

$\frac{\tan^2 x}{2}\Big|_0^{\pi/3}$

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$\int_0^1 \frac{x}{x^2-4}\,dx$

$u=x^2-4$

$x=1$ $u=1^2-4=-3$

$x=0$ $u=0^2-4=-4$

$\frac{du}{dx}=2x$

$du=2x\,dx$

$\frac{du}{2x}=dx$

$\int \frac{\cancel{x}}{u}\frac{du}{\cancel{2x}}$

$\frac{1}{2}\int\frac{1}{u}du$

$\frac{1}{2}\ln|u|\Big|_{-4}^{-3}$

$\frac{1}{2}(\ln 3-\ln 4)=\frac{1}{2}\ln\frac{3}{4}=\ln\sqrt{\frac{3}{4}}$

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